

GoSam: A program for automated one-loop Calculations

G. Cullen

Deutsches Elektronen-Synchrotron DESY, Zeuthen, Germany

N. Greiner, G. Heinrich¹, T. Reiter

Max-Planck-Institute for Physics, Munich, Germany

G. Luisoni

Institute for Particle Physics Phenomenology, University of Durham, UK

P. Mastrolia

Max-Planck Institute for Physics, Munich, Germany;
Dipartimento di Fisica, Università di Padova, Italy

G. Ossola

New York City College of Technology, City University of New York

F. Tramontano

CERN, Geneva, Switzerland

Abstract. The program package GoSAM is presented which aims at the automated calculation of one-loop amplitudes for multi-particle processes. The amplitudes are generated in terms of Feynman diagrams and can be reduced using either D-dimensional integrand-level decomposition or tensor reduction, or a combination of both. GoSAM can be used to calculate one-loop corrections to both QCD and electroweak theory, and model files for theories Beyond the Standard Model can be linked as well. A standard interface to programs calculating real radiation is also included. The flexibility of the program is demonstrated by various examples.

1. Introduction

Precise theory predictions play an important role in the analysis and the interpretation of collider physics data, in the search for new particles as well as to constrain model parameters at a later stage. Therefore it is desirable to have predictions at next-to-leading order (NLO) in perturbation theory as a standard at the LHC. However, this can only be achieved if a level of

¹ Speaker; Talk given at the International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT), Uxbridge, London, September 2011.

automation for NLO predictions is reached which comes close to the one we have for leading order tools.

The need for an automation of NLO calculations has been noticed some time ago and lead to public programs like FeynArts [1] and QGraf [2] for diagram generation and FormCalc/LoopTools [3] and GRACE [4] for the automated calculation of NLO corrections. However, calculations of one-loop amplitudes with more than four external legs were still tedious case-by-case calculations. Only very recently, conceptual and technical advances in multi-leg one-loop calculations opened the door to the possibility of an *automated* generation and evaluation of multi-leg one-loop amplitudes. As a consequence, public NLO tools containing a collection of hard-coded individual processes, like e.g. MCFM [5, 6], VBFNLO [7, 8, 9], MC@NLO [10, 11], POWHEG-Box [12, 13], POWHEL [14, 15, 16], are now being complemented by flexible automated tools [17, 18, 19, 20, 21] which have as their aim that basically any process which may turn out to be important for the comparison of LHC findings to theory can be evaluated at NLO accuracy.

In this talk, we present the program package GOSAM [17] which allows the automated calculation of one-loop amplitudes for multi-particle processes. Amplitudes are expressed in terms of Feynman diagrams, where the integrand is generated analytically using QGRAF [2], FORM [22], spinney [23] and haggies [24]. The individual program tasks are steered via python scripts, while the user only needs to edit an “input card” to specify the details of the process to be calculated, and launch the generation of the source code and its compilation, without having to worry about internal details of the code.

The program offers the option to use different reduction techniques: either the unitarity-based integrand reduction [25, 26] as implemented in SAMURAI [27] or traditional tensor reduction as implemented in golem95C [28, 29] interfaced through tensorial reconstruction at the integrand level [30], or a combination of both. It can be used to calculate one-loop corrections within both QCD and electroweak theory. Beyond the Standard Model theories can be interfaced using FeynRules [31] or LanHEP [32]. The Binoth Les Houches interface [33] to programs providing the real radiation contributions is also included.

2. Theoretical framework

2.1. Generation of the diagrams

For the diagram generation we use the program QGRAF [2] and supplement it by adding another level of analysing and filtering the diagrams, written in Python. This allows for example to drop diagrams whose colour factor turns out to be zero, or to determine the signs for diagrams with Majorana fermions.

Within our framework, QGRAF generates three sets of output files: an expression for each diagram to be processed with FORM [22], Python code to draw all diagrams, and Python code to compute the properties of each diagram. The information about the model is either read from the built-in Standard Model file or is generated from a user defined LanHEP [32] or Universal FeynRules Output (UFO) [31] file.

The program also produces a L^AT_EX file which contains graphical representations of all diagrams as well as a summary of the helicity and colour basis used.

2.2. Code generation

The amplitude is generated in terms of algebraic expressions based on Feynman diagrams and then processed with a FORM program, using spinney [23] for the spinor algebra. In GOSAM we have implemented the 't Hooft-Veltman scheme (HV) and dimensional reduction (DRED). In both schemes all external vectors (momenta and polarisation vectors) are kept in four dimensions, while internal vectors are kept in the n -dimensional vector space ($n = 4 - 2\epsilon$). We adopt the conventions used in [23], where \hat{k} denotes the four-dimensional projection of an n -dimensional

vector k . The $(n-4)$ -dimensional orthogonal projection is denoted by \tilde{k} . For the loop momentum q we introduce the symbol $\mu^2 = -\tilde{q}^2$, such that

$$q^2 = \hat{q}^2 + \tilde{q}^2 = \hat{q}^2 - \mu^2. \quad (1)$$

To prepare the numerator functions of the one-loop diagrams for their numerical evaluation, we separate the symbol μ^2 and dot products involving the momentum \hat{q} from all other factors. All subexpressions which do not depend on either \hat{q} or μ^2 are substituted by abbreviations, which are evaluated only once per phase space point. Each of the two parts is then processed by `haggies` [24], which generates optimised `Fortran95` code for the numerical evaluation. For each diagram we generate an interface to `SAMURAI` [27], `golem95C` [29] and/or `PJFRY` [34].

Our standard choice for the reduction is to use `SAMURAI` [27], which provides a very fast and stable reduction in a large part of the phase space. Furthermore, `SAMURAI` reports to the client code if the quality of the reconstruction of the numerator suffices the numerical requirements. In `GoSAM` we use this information to trigger an alternative reduction with either `golem95C` [29] or `PJFRY` [34] whenever these reconstruction tests fail. This combination of on-shell techniques and traditional tensor reduction is achieved using tensorial reconstruction at the integrand level [30]. The tensorial reconstruction not only can cure numerical instabilities, but in some cases also can reduce the computational cost of the reduction. Since the reconstructed numerator is typically of a form where kinematics and loop momentum dependence are already separated, the use of a reconstructed numerator tends to be faster than the original procedure, in particular in cases with a large number of legs and low rank.

2.3. Rational terms

Terms containing the symbols μ^2 or ε in the numerator of the integrands can lead to a so-called R_2 term [35], which contributes to the rational part of the amplitude. We generate the R_2 part along with all other contributions without the need to separate the different parts. In addition, we provide an *implicit* and an *explicit* construction of the R_2 terms, using the fact that there are two ways of splitting the numerator function:

$$\mathcal{N}(\hat{q}, \mu^2, \varepsilon) = \mathcal{N}_0(\hat{q}, \mu^2) + \varepsilon \mathcal{N}_1(\hat{q}, \mu^2) + \varepsilon^2 \mathcal{N}_2(\hat{q}, \mu^2) \quad (2)$$

or, alternatively,

$$\mathcal{N}(\hat{q}, \mu^2, \varepsilon) = \hat{\mathcal{N}}(\hat{q}) + \tilde{\mathcal{N}}(\hat{q}, \mu^2, \varepsilon). \quad (3)$$

The implicit construction uses the splitting of Eq. (2) and treats all three numerator functions \mathcal{N}_i on equal grounds. Each of the three terms is reduced separately in a numerical reduction and the Laurent series of the three results are added up taking into account the powers of ε .

The explicit construction of R_2 is based on the assumption that each term in $\tilde{\mathcal{N}}$ in Eq. (3) contains at least one power of μ^2 or ε . The expressions for those integrals are relatively simple and known explicitly. Hence, the part of the amplitude which originates from $\tilde{\mathcal{N}}$ is computed analytically whereas the purely four-dimensional part $\hat{\mathcal{N}}$ is passed to the numerical reduction.

In the program, possible options for R_2 are `r2=implicit,explicit,off` and `only`. Using `r2=only` discards everything but the R_2 term and puts `GoSAM` in the position of providing R_2 terms to supplement other codes which work entirely in four dimensions.

2.4. Conventions

To be specific, we consider the case where the user wants to compute QCD corrections. In the case of electroweak corrections, the analogous conventions apply except that the strong coupling g_s is replaced by e . In the QCD case, the tree-level matrix element squared can be written as

$$|\mathcal{M}|_{\text{tree}}^2 = \mathcal{A}_0^\dagger \mathcal{A}_0 = (g_s)^{2b} \cdot a_0, \quad (4)$$

where $b = 0$ is also possible. The matrix element at one-loop level, i.e. the interference term between tree-level and one-loop, can be written as

$$|\mathcal{M}|_{1\text{-loop}}^2 = \mathcal{A}_1^\dagger \mathcal{A}_0 + \mathcal{A}_0^\dagger \mathcal{A}_1 = 2 \cdot \Re(\mathcal{A}_0^\dagger \mathcal{A}_1) = \frac{\alpha_s(\mu)}{2\pi} \frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)} (g_s)^{2b} \left[c_0 + \frac{c_{-1}}{\varepsilon} + \frac{c_{-2}}{\varepsilon^2} + \mathcal{O}(\varepsilon) \right]. \quad (5)$$

A call to the subroutine `samplitude` returns an array consisting of the four numbers $(a_0, c_0, c_{-1}, c_{-2})$ in this order. The average over initial state colours and helicities is included in the default setup. Renormalisation is included depending on the options chosen by the user, for a more detailed description we refer to [17]. In cases where the process is loop induced, i.e. the tree level amplitude is absent, the program returns the values for $\mathcal{A}_1^\dagger \mathcal{A}_1$

$$\left(\frac{\alpha_s(\mu)}{2\pi} \frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)} \right)^2$$

has been extracted.

3. Installation and Usage

3.1. Installation

The user can download the code GOSAM either as a tar-ball or from the subversion repository at

<http://projects.hepforge.org/gosam/> .

The build process and installation of GOSAM is controlled by `Python Distutils`, while the build process for the libraries `SAMURAI` and `golem95C` is controlled by `Autotools`. To install GOSAM, the user needs to run

```
python setup.py install --prefix MYPATH
```

If `MYPATH` is different from the system default, the environment variables `PATH`, `LD_LIBRARY_PATH` and `PYTHONPATH` might have to be set accordingly. For more details we direct the user to the GOSAM reference manual coming with the code.

The program is designed to run in any modern Linux/Unix environment; we expect that `Python` (≥ 2.6), `Java` (≥ 1.5) and `Make` are installed on the system. Furthermore, a Fortran 95 compiler is required in order to compile the generated code.

On top of a standard Linux environment, the programs `FORM` [22], version ≥ 3.3 , and `QGRAF` [2] need to be installed on the system. Further, at least one of the libraries `SAMURAI` [27] and `golem95C` [29] needs to be present at compile time of the generated code. For the user's convenience we have prepared a package containing `SAMURAI` and `golem95C` together with the integral libraries `OneLOop` [36], `QCDLoop` [37] and `FF` [38]. The package `gosam-contrib-1.0.tar.gz` containing all these libraries is available for download from <http://projects.hepforge.org/gosam/>.

3.2. Usage

In order to generate the code for a process, the user needs to prepare an input file, called *process card* in the following, which contains

- process specific information, such as a list of initial and final state particles, their helicities (optional) and the order of the coupling constants;
- scheme specific information and approximations, such as the regularisation and renormalisation schemes, the underlying model, masses and widths which are set to zero, the selection of subsets of diagrams, etc;

Table 1. Part of the process card defining the initial and final state and the type and order of the coupling at LO and NLO

```
process_path= qqgz
in=          u,u~
out=         g, e-, e+
helicities=  + - + - +, + - - - +, - + + - +, - + - - +
order=       QCD, 1, 3
```

- system specific information, such as paths to programs and libraries or compiler options;
- optional information for optimisations which control the code generation.

For example, to compute the QCD corrections for the process $u\bar{u} \rightarrow gZ^0 \rightarrow g e^- e^+$, the first part of the process card looks as shown in Table 1: The first line defines the (relative) path to the directory where the process files will be generated. GOSAM expects that this directory has already been created. Lines 2 and 3 define the initial and final state of the process in terms of field names, which are defined in the model file. Besides the field names one can also use PDG codes [39, 40] instead. For more details we refer to [17] and the reference manual.

3.3. Code generation

If the process card is called `gosam.in`, it can be invoked by `gosam.py`:

```
mkdir qqgz
gosam.py gosam.in
cd qqgz
```

All further steps are controlled by the generated make files; in order to generate and compile all files relevant for the matrix element one needs to invoke

```
make compile
```

The generated code can be tested with the program `matrix/test.f90`. The following sequence of commands will compile and run the test program:

```
cd matrix
make test.exe
./test.exe
```

The last lines of the program output should look as follows.

```
#          LO:  0.3450350717601E-06
# NLO, finite part  -10.77604823456547
# NLO, single pole  -19.98478948141949
# NLO, double pole  -5.666666665861926
# IR,  single pole  -19.98478948439310
# IR,  double pole  -5.666666666666666
```

The printed numbers are, in this order, a_0 , c_0/a_0 , c_{-1}/a_0 , c_{-2}/a_0 and the pole parts calculated from the infrared insertion operator [41, 42]. One can also generate a pictorial representation of all generated diagrams using the command

```
make doc
```

which generates the file `doc/process.ps`

3.4. Interfacing the code

3.4.1. Using the BLHA Interface The so-called *Binoth Les Houches Accord* (BLHA) [33] defines an interface for a standardized communication between one-loop programs (OLP) and Monte Carlo (MC) tools. GoSAM can act as an OLP in the framework of the BLHA, such that the calculation of complete cross sections is straightforward.

In general, the MC writes an order file, called for example `olp_order.lh`, and invokes the script `gosam.py` as follows:

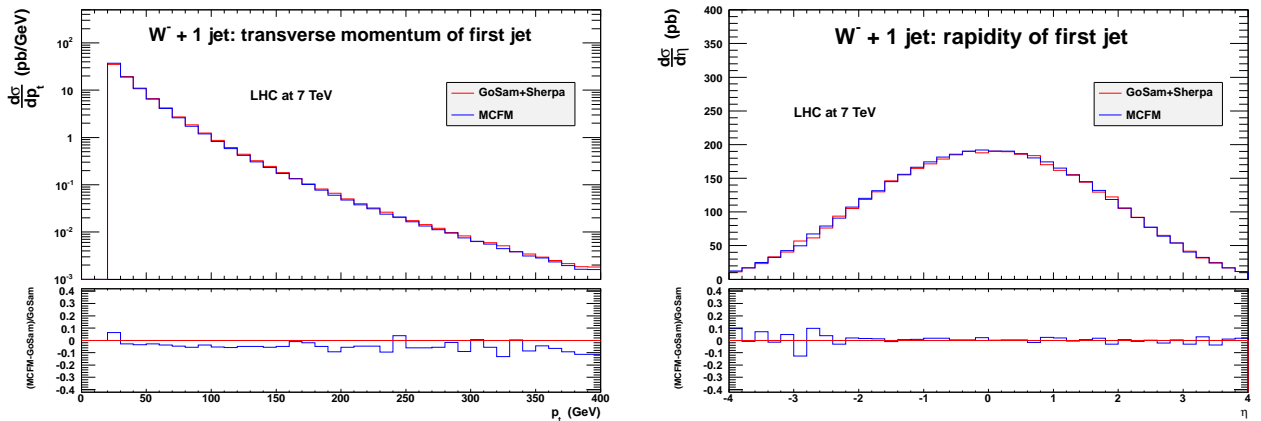
```
gosam.py --olp olp_order.lh
```

The invocation of `gosam.py` generates a set of files which can be compiled with a generated make file. The BLHA routines are defined in the Fortran module `olp_module` but can also be accessed from C programs.

3.4.2. Using External Model Files GoSAM can also make use of model files generated by either Feynrules [43] in the UFO format [31] or by LanHEP [32]. The particles can be specified by their PDG code. Details about how to import the model files are described in the GoSAM reference manual. Precompiled `MSSM_UFO` and `MSSM_LHEP` files can also be found in the subdirectory `examples/model`. The `examples` directory also contains examples where UFO or LanHEP model files are imported.

4. Examples

The code generated by GoSAM has been compared to a considerable number of processes available in the literature, as listed in Table 2. Many of these processes are also included as examples in the code, including reference values.



(a) Transverse momentum of the leading jet for $W^- + \text{jet}$ production at LHC with $\sqrt{s} = 7$ TeV. (b) Pseudorapidity of the leading jet for $W^- + \text{jet}$ production at LHC with $\sqrt{s} = 7$ TeV.

Figure 1. NLO calculation of $W^- + \text{jet}$ production at LHC using GoSAM interfaced with SHERPA via the Binoth Les Houches interface and compared to MCFM.

As an example for the *Binoth Les Houches Accord* interface of GoSAM we present results for the QCD corrections to $W^- + 1$ jet production, obtained by linking GoSAM with SHERPA [59]. Furthermore, SHERPA offers the possibility to match NLO calculations with a parton shower [60, 61]. Results for the transverse momentum and rapidity distribution of the leading jet are shown in Figs. 1(a) and 1(b). The comparison with MCFM [5, 6] shows perfect agreement.

process	checked with Ref.
$e^+e^- \rightarrow u\bar{u}$	[44]
$e^+e^- \rightarrow t\bar{t}$	[45, 46], own analytic calculation
$u\bar{u} \rightarrow d\bar{d}$	[47, 19]
$gg \rightarrow gg$	[48]
$gg \rightarrow gZ$	[49]
$d\bar{d} \rightarrow t\bar{t}$	[19], MCFM [5, 50]
$gg \rightarrow t\bar{t}$	[19], MCFM [5, 50]
$bg \rightarrow Hb$	[51, 19]
$\gamma\gamma \rightarrow \gamma\gamma$	[52]
$u\bar{d} \rightarrow e^-\bar{\nu}_e$	[19]
$u\bar{d} \rightarrow e^-\bar{\nu}_e g$	[19]
$e^+e^- \rightarrow e^+e^-\gamma$ (QED)	[53]
$pp \rightarrow Ht\bar{t}$	[19]
$pp \rightarrow W^+W^+jj$	[54, v3]
$pp \rightarrow W^\pm j$ (QCD corr.)	MCFM [5, 50]
$pp \rightarrow W^\pm j$ (EW corr.)	for IR poles: Eqs. (67),(70) of [55], [56]
$pp \rightarrow b\bar{b}b\bar{b}$	[57, 58]
$pp \rightarrow W^+W^-b\bar{b}$	[18, 19]
$u\bar{u} \rightarrow t\bar{t}b\bar{b}$	[18, 19]
$gg \rightarrow t\bar{t}b\bar{b}$	[18, 19]
$u\bar{d} \rightarrow W^+ggg$	[18]

Table 2. Processes for which GoSAM has been compared to the literature.

5. Conclusions

We have presented the program GoSAM which can produce code for the evaluation of one-loop matrix elements for multi-particle processes in an automated way. The program is publicly available at <http://projects.hepforge.org/gosam/> and can be used to calculate one-loop amplitudes within QCD, electroweak theory, or other models which can be imported via an interface to LanHEP or FeynRules. Monte Carlo programs for the real radiation can be easily linked via the interface defined by the *Binoth Les Houches Accord*.

The amplitudes are generated in terms of Feynman diagrams and can be reduced by unitarity based reduction at integrand level or traditional tensor reduction, or a combination of the two approaches. This feature makes the program extremely robust against numerical instabilities. The user can choose among different libraries for the master integrals, and the setup is such that other libraries can be linked easily.

The calculation of the rational terms can proceed either together with the same numerical reduction as the rest of the amplitude, or before any reduction, using analytic information on the integrals which can potentially give rise to a rational part. Moreover, the GoSAM generator can produce code for processes which include unstable particles, i.e. intermediate states with complex masses.

GoSAM is very well suited for the automated matching of Monte Carlo programs to NLO virtual amplitudes, and therefore can be used as a module to produce differential cross sections for multi-particle processes which can be compared directly to experiment. Thus GoSAM can contribute to the goal of automating NLO corrections to an extent where NLO tools become a standard for data analysis at the LHC.

Acknowledgments

G.C. and G.L. were supported by the British Science and Technology Facilities Council (STFC). The work of G.C. was supported by DFG Sonderforschungsbereich Transregio 9, Computergestützte Theoretische Teilchenphysik. N.G. was supported in part by the U.S. Department of Energy under contract No. DE-FG02-91ER40677. P.M. and T.R. were supported by the Alexander von Humboldt Foundation, in the framework of the Sofja Kovaleskaja Award Project “Advanced Mathematical Methods for Particle Physics”, endowed by the German Federal Ministry of Education and Research. The work of G.O. was supported in part by the National Science Foundation under Grant PHY-0855489 and PHY-1068550. The research of F.T. is supported by Marie-Curie-IEF, project: “SAMURAI-Apps”. We also acknowledge the support of the Research Executive Agency (REA) of the European Union under the Grant Agreement number PITN-GA-2010-264564 (LHCPhenoNet).

References

- [1] Hahn T 2001 *Comput.Phys.Commun.* **140** 418–431 (*Preprint hep-ph/0012260*)
- [2] Nogueira P 1993 *J.Comput.Phys.* **105** 279–289
- [3] Hahn T and Perez-Victoria M 1999 *Comput.Phys.Commun.* **118** 153–165 (*Preprint hep-ph/9807565*)
- [4] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T *et al.* 2006 *Phys.Rept.* **430** 117–209 (*Preprint hep-ph/0308080*)
- [5] Campbell J M and Ellis R 1999 *Phys.Rev.* **D60** 113006 (*Preprint hep-ph/9905386*)
- [6] Campbell J M, Ellis R and Williams C 2011 *JHEP* **1107** 018 (*Preprint 1105.0020*)
- [7] Arnold K, Bahr M, Bozzi G, Campanario F, Englert C *et al.* 2009 *Comput.Phys.Commun.* **180** 1661–1670 (*Preprint 0811.4559*)
- [8] Arnold K, Bellm J, Bozzi G, Brieg M, Campanario F *et al.* 2011 (*Preprint 1107.4038*)
- [9] Campanario F 2011 *JHEP* **1110** 070 (*Preprint 1105.0920*)
- [10] Frixione S and Webber B R 2002 *JHEP* **0206** 029 (*Preprint hep-ph/0204244*)
- [11] Frixione S, Stoeckli F, Torrielli P, Webber B R and White C D 2010 (*Preprint 1010.0819*)
- [12] Frixione S, Nason P and Oleari C 2007 *JHEP* **0711** 070 (*Preprint 0709.2092*)
- [13] Alioli S, Nason P, Oleari C and Re E 2010 *JHEP* **1006** 043 (*Preprint 1002.2581*)
- [14] Kardos A, Papadopoulos C and Trocsanyi Z 2011 *Phys.Lett.* **B705** 76–81 (*Preprint 1101.2672*)
- [15] Garzelli M, Kardos A, Papadopoulos C and Trocsanyi Z 2011 *Europhys.Lett.* **96** 11001 (*Preprint 1108.0387*)
- [16] Kardos A, Papadopoulos C and Trocsanyi Z 2011 (*Preprint 1111.0610*)
- [17] Cullen G, Greiner N, Heinrich G, Luisoni G, Mastrolia P *et al.* 2011 (*Preprint 1111.2034*)
- [18] van Hameren A, Papadopoulos C and Pittau R 2009 *JHEP* **0909** 106 (*Preprint 0903.4665*)
- [19] Hirschi V, Frederix R, Frixione S, Garzelli M V, Maltoni F *et al.* 2011 *JHEP* **1105** 044 (*Preprint 1103.0621*)
- [20] Bevilacqua G, Czakon M, Garzelli M, van Hameren A, Kardos A *et al.* 2011 (*Preprint 1110.1499*)
- [21] Hahn T, *Talk given at the International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT)*, Uxbridge, London, September 2011.
- [22] Vermaseren J 2000 (*Preprint math-ph/0010025*)
- [23] Cullen G, Koch-Janusz M and Reiter T 2011 *Comput.Phys.Commun.* **182** 2368–2387 (*Preprint 1008.0803*)
- [24] Reiter T 2010 *Comput.Phys.Commun.* **181** 1301–1331 (*Preprint 0907.3714*)
- [25] Ossola G, Papadopoulos C G and Pittau R 2007 *Nucl.Phys.* **B763** 147–169 (*Preprint hep-ph/0609007*)
- [26] Ellis R, Giele W T, Kunszt Z and Melnikov K 2009 *Nucl.Phys.* **B822** 270–282 (*Preprint 0806.3467*)
- [27] Mastrolia P, Ossola G, Reiter T and Tramontano F 2010 *JHEP* **1008** 080 (*Preprint 1006.0710*)
- [28] Binoth T, Guillet J P, Heinrich G, Pilon E and Reiter T 2009 *Comput.Phys.Commun.* **180** 2317–2330 (*Preprint 0810.0992*)
- [29] Cullen G, Guillet J, Heinrich G, Kleinschmidt T, Pilon E *et al.* 2011 *Comput.Phys.Commun.* **182** 2276–2284 (*Preprint 1101.5595*)
- [30] Heinrich G, Ossola G, Reiter T and Tramontano F 2010 *JHEP* **1010** 105 (*Preprint 1008.2441*)
- [31] Degrande C, Duhr C, Fuks B, Grellscheid D, Mattelaer O *et al.* 2011 (*Preprint 1108.2040*)
- [32] Semenov A 2010 (*Preprint 1005.1909*)
- [33] Binoth T, Boudjema F, Dissertori G, Lazopoulos A, Denner A *et al.* 2010 *Comput.Phys.Commun.* **181** 1612–1622 dedicated to the memory of, and in tribute to, Thomas Binoth, who led the effort to develop this proposal for Les Houches 2009 (*Preprint 1001.1307*)
- [34] Fleischer J and Riemann T 2011 *Phys.Rev.* **D83** 073004 (*Preprint 1009.4436*)
- [35] Ossola G, Papadopoulos C G and Pittau R 2008 *JHEP* **0805** 004 (*Preprint 0802.1876*)
- [36] Van Hameren A 2011 *Comput.Phys.Commun.* **182** 2427–2438 (*Preprint 1007.4716*)

- [37] Ellis R K and Zanderighi G 2008 *JHEP* **02** 002 (*Preprint* 0712.1851)
- [38] Van Oldenborgh G and Vermaseren J 1990 *Z.Phys.* **C46** 425–438
- [39] Yost G *et al.* (Particle Data Group) 1988 *Phys.Lett.* **B204** 1–486
- [40] Caso C *et al.* (Particle Data Group) 1998 *Eur.Phys.J.* **C3** 1–794
- [41] Catani S and Seymour M 1997 *Nucl.Phys.* **B485** 291–419 (*Preprint* hep-ph/9605323)
- [42] Catani S, Dittmaier S and Trocsanyi Z 2001 *Phys.Lett.* **B500** 149–160 (*Preprint* hep-ph/0011222)
- [43] Christensen N D and Duhr C 2009 *Comput. Phys. Commun.* **180** 1614–1641 (*Preprint* 0806.4194)
- [44] Ellis R, Stirling W and Webber B 1996 *Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol.* **8** 1–435
- [45] Jersak J, Laermann E and Zerwas P 1982 *Phys.Rev.* **D25** 1218
- [46] Catani S, Dittmaier S, Seymour M H and Trocsanyi Z 2002 *Nucl.Phys.* **B627** 189–265 (*Preprint* hep-ph/0201036)
- [47] Ellis R and Sexton J 1986 *Nucl.Phys.* **B269** 445
- [48] Binoth T, Guillet J and Heinrich G 2007 *JHEP* **0702** 013 (*Preprint* hep-ph/0609054)
- [49] van der Bij J and Glover E 1989 *Nucl.Phys.* **B313** 237
- [50] Campbell J M and Ellis R K 2000 *Phys. Rev.* **D62** 114012 (*Preprint* hep-ph/0006304)
- [51] Campbell J M, Ellis R, Maltoni F and Willenbrock S 2003 *Phys.Rev.* **D67** 095002 (*Preprint* hep-ph/0204093)
- [52] Gounaris G, Porfyriadis P and Renard F 1999 *Eur.Phys.J.* **C9** 673–686 (*Preprint* hep-ph/9902230)
- [53] Actis S, Mastrolia P and Ossola G 2010 *Phys.Lett.* **B682** 419–427 (*Preprint* 0909.1750)
- [54] Melia T, Melnikov K, Rontsch R and Zanderighi G 2010 *JHEP* **1012** 053 (*Preprint* 1007.5313)
- [55] Kuhn J H, Kulesza A, Pozzorini S and Schulze M 2008 *Nucl.Phys.* **B797** 27–77 (*Preprint* 0708.0476)
- [56] Gehrmann T and Greiner N 2010 *JHEP* **12** 050 (*Preprint* 1011.0321)
- [57] Binoth T, Greiner N, Guffanti A, Reuter J, Guillet J P *et al.* 2010 *Phys.Lett.* **B685** 293–296 (*Preprint* 0910.4379)
- [58] Greiner N, Guffanti A, Reiter T and Reuter J 2011 *Phys.Rev.Lett.* **107** 102002 (*Preprint* 1105.3624)
- [59] Gleisberg T, Höche S, Krauss F, Schonherr M, Schumann S *et al.* 2009 *JHEP* **0902** 007 (*Preprint* 0811.4622)
- [60] Höche S, Krauss F, Schonherr M and Siegert F 2011 *JHEP* **1104** 024 (*Preprint* 1008.5399)
- [61] Höche S, Krauss F, Schonherr M and Siegert F 2011 (*Preprint* 1111.1220)